RECENT ADVANCES IN NRQCD *

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We discuss recent theoretical developments concerning the description of the production and decay of heavy quarks and colored scalars in the framework of nonrelativistic QCD.

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1. Threshold Physics at the ILC

The e^+e^- center-of-mass (c.m.) energy at a linear collider (LC) can be very precisely monitored, allowing for an accurate exploration of the threshold regime. The top-quark mass can be determined from a measurement of $\sigma(e^+e^- \to Z^*, \gamma^* \to t\bar{t})$ line shape at a LC operating at c.m. energies around the $t\bar{t}$ threshold ($\sqrt{q^2} \sim 350$ GeV). The rise of the cross section with increasing c.m. energy is directly related to the mass of the top quark. Assuming a total integrated luminosity of 300 fb^{-1}, LC simulations of a threshold scan of the top-antitop total cross section have demonstrated that experimental uncertainties below 100 MeV for the top-quark mass determination can be obtained [1; 2], even when beam effects, which lead to some smearing of the effective c.m. energy, are taken into account. If the normalization of the cross section line shape is well under control, it is possible to determine the strong coupling, the total top quark width and, if the Higgs boson is light, the top Yukawa coupling. In view of the accuracy obtainable at the LC the theoretical uncertainties for the total cross section should be lowered to a level of a few percent [3; 4]. A precise knowledge of the top mass would also improve the analysis of electroweak precision observables and put indirect constraints on New Physics. It has

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been shown [5] that an accuracy of 100 MeV on the top quark mass would allow to perform stringent internal consistency checks of the SM and of some scenarios of Supersymmetry (SUSY).

Threshold studies would also be feasible for squarks at a next LC. Many models of SUSY predict that, due to large mixing, the lightest squark could correspond to one of the mass eigenstates of the third generation with $m_{\tilde{q}} < 500$ GeV, thus allowing for the production of stop pairs at a future e^+e^- LC operating below 1 TeV. A "threshold scan" of the total cross section line-shape at such a facility will yield precise measurements of the stop mass, lifetime and couplings [6], in close analogy to the program carried out in threshold studies for the top-antitop threshold [2].

2. Theoretical Status of $t\bar{t}$ Production

Close to threshold the top quark pairs are produced with small velocities $v \ll 1$ in the c.m. frame. Therefore the relevant physical scales governing the top-antitop dynamics are the top quark mass m_t, the relative three-momentum $\boldsymbol{p} \sim m_t\, v$ and the top quark nonrelativistic kinetic energy $E \sim m_t\, v^2$. Since the ratios of the three scales can arise in matrix elements, the cross section cannot be calculated using the standard QCD expansion in the strong coupling α_s. The best known indication of the latter comes from the well-known "Coulomb singularity", which shows up as a singular $(\alpha_s/v)^n$ behaviour in the $v \to 0$ limit of the $t\bar{t}$ production amplitude at the n-loop order in perturbative QCD. The proper expansion scheme for the $t\bar{t}$ threshold region is a double expansion in both α_s and v, and one has to use the parametric counting $\alpha_s \sim v \ll 1$ to identify all effects contributing to a certain order of approximation. In this *fixed-order* expansion the leading order (LO) contributions correspond to terms in the total cross section proportional to $v(\alpha_s/v)^n$, $(n = 0, \ldots, \infty)$, next-to-leading (NLO) terms are proportional to $v(\alpha_s/v)^n \times [\alpha_s, v]$ and so on. At LO the total cross section is proportional to the absorptive part of the Green function of a Schrödinger equation containing the static QCD-potential [7]. Higher order corrections in this expansion are rigorously implemented employing the concept of nonrelativistic effective quantum field theories, first proposed by Caswell and Lepage [8]. In this scheme, the original QCD Lagrangian is reformulated in terms of an effective nonrelativistic Lagrangian called "Nonrelativistic Quantum Chromodynamics" (NRQCD) by using the hierarchy $m_t \gg \boldsymbol{p} \gg E$, which allows to separate short-distance physics at the "hard" scale of order the heavy quark mass from long-distance physics at the nonrelativistic scales $\boldsymbol{p}$ and E. The hard-momentum effects are encoded as Wilson coefficients of the operators in the effective Lagrangian. Operators with increasing dimension are introduced in the effective theory to include the effects of higher orders in the nonrelativistic expansion, but only a finite number is needed for a given precision. The NRQCD factorization properties were used in a number of NNLO calculations [9; 10] of the total $t\bar{t}$ production cross section.

A common feature of the *fixed-order* calculations is that the running from the hard scale m down to the nonrelativistic scales was not taken into account. At NNLO NRQCD matrix elements and Wilson coefficients involve logarithms of ratios of the hard scale and the nonrelativistic scales, which in the case of top quark pair production close to threshold can be sizeable (for example $\alpha_s(m_t)\log[m_t/E] \simeq 0.8$ for $v \simeq 0.15$). Considering the parametric counting $\alpha_s \log v \sim 1$ introduces a modified expansion scheme for the size of the terms contributing to the total cross section, where the dominant contribution, proportional to $(\alpha_s/v)^n \sum_i(\alpha_s \log v)^i$, $(n = 0, \ldots \infty)$, is called leading-logarithmic order (LL). A number of different versions of NRQCD [11; 12], each of which aiming (in principle) on applications in different physical situations, allow for renormalization group improved calculations (see also Ref. [13] for recent reviews on the field). The EFT vNRQCD ("velocity"NRQCD) [12; 14] has been designed for predictions at the $t\bar{t}$ threshold. It treats the case $m_t \gg \boldsymbol{p} \gg E > \Lambda_{\rm QCD}$, i.e. all physical scales are perturbative, but also has the correlation $E_t = \boldsymbol{p}_t^2/m_t$ built in at the field theoretical level. QCD effects for the $t\bar{t}$ total cross section up to the NNLL order computed in this framework [4] significantly reduce the size and scale dependence to yield a 6% theoretical uncertainty.

The effective Lagrangian for the scalar version of vNRQCD, which describes the nonrelativistic interaction between pairs of colored scalars and provides the needed ingredients for a summation of QCD effects at NLL order, has been given recently [15].

3. Finite Width and Electroweak Effects

Up to now, no systematic and complete treatment of electroweak effects in the total cross section for top quark pair production has been achieved beyond the LO approximation. The large top width, being of the same order than the nonrelativistic energy, is essential in the description of the $t\bar{t}$ threshold dynamics. It was shown [7] that in the nonrelativistic limit the top-quark width can be consistently implemented by the replacement $E \to E + i\Gamma_t$ in the results for the total cross section for stable top quarks. Although this replacement rule can accommodate some of the NLO and

NNLO electroweak corrections, a coherent treatment at the conceptual level requires the use of an extended NRQCD effective theory formalism.

In the NRQCD/vNRQCD framework as long as one is not interested in any differential information of the top decay, finite lifetime corrections to the total cross section can be regarded as short-distance information to be encoded in the Wilson coefficients of the NRQCD Lagrangian and the NRQCD currents. As the particles involved in the electroweak corrections can be lighter than the top quark they can lead to nonzero imaginary parts in the matching conditions. These electroweak absorptive parts render the NRQCD Lagrangian non-hermitian, but the total cross section can still be obtained from the imaginary part of the $e^+e^- \to e^+e^-$ forward scattering amplitude by virtue of the optical theorem and the unitarity of the underlying theory, in complete analogy to the treatment of the inelastic processes in quantum mechanics where particle decay and absorption are implemented through potentials with complex coefficients. An important feature of the effective theory treatment of electroweak effects is that resonant and non-resonant contributions in NRQCD amplitudes can be systematically separated if the scaling relation $v \sim \alpha_s \sim \alpha^{1/2}$ is used. The latter is justified because numerically the top width is approximately equal to the typical top kinetic energy $\Gamma_t \sim m_t\alpha \sim E_{\rm kin} \sim m_t\alpha_s^2$. Including the non-resonant background diagrams leading to the same final states as those of top decay in the matching calculations is necessary in order to maintain gauge invariance.

In this approach the NNLL matching conditions accounting for the absorptive parts related to the $bW^+\bar{b}W^-$ final state were derived in [16] and shown to amount numerically to several percent. A very interesting new conceptual aspect of these corrections is that they have UV divergences that arise from the high energy behaviour of the $t\bar{t}$ effective theory phase space integration. The phase-space in the full theory is cut-off by the large top mass, but it extends to infinity in NRQCD where we have taken the limit $m_t \to \infty$ [a]. The divergences show up only when $\Gamma_t \neq 0$, as a finite width generates a distribution for the top invariant mass and thus allows for arbitrary large momenta in the nonrelativistic phase space integration. Similar divergences had already been noted in the QCD NNLL relativistic corrections to the S-wave zero distance Green function if the unstable propagator was used [4; 9] as well as in the leading order P-wave

[a]Instead of using the optical theorem to obtain the total cross section from the forward scattering amplitude one can integrate over the phase-space explicitly with cuts and these UV divergences obviously do not arise

zero distance Green function which accounts for $t\bar{t}$ production through the Z-exchange [4]. The NNLL divergences renormalize $(e^+e^-)(e^+e^-)$ operators that contribute to the total cross section through the imaginary parts of their Wilson coefficients. The running induced in these Wilson coefficients by the divergences thus represents a NLL effect to the total cross section [16] but their matching conditions at the hard scale are presently unknown.

Scalar particles are produced at leading order in the nonrelativistic expansion in a P-wave state. In these systems the phase-space divergences constitute a more severe problem as they show up already in the leading order Green's function. A phenomenologically well motivated example is pair production of SUSY partners of the quarks at threshold, which has been studied at LO in several works [17] using a semi-phenomenological solution in order to deal with the phase-space UV divergences.

4. Nonrelativistic Currents with General Quantum Numbers

There are a number of issues concerning the consistent formulation of nonrelativistic interpolating currents which describe color singlet heavy quark-antiquark and squark-antisquark pair production for general quantum numbers in $n = 3-2\epsilon$ dimensions. These involve in particular the generalization of spherical harmonics in n spatial dimensions and the role of evanescent operator structures (*i.e.* that vanish as $\epsilon \to 0$) for the description of the nonrelativistic spin. The latter have been addressed in detail in a recent work [18] and we briefly comment on them here.

Let us discuss first the spin singlet currents with arbitrary angular momentum L (${}^{2S+1}L_J = {}^1L_L$). The generic structure of the production currents with total spin zero is $\psi^\dagger_{\boldsymbol{p}}(x)\,\Gamma(\boldsymbol{p})\,\tilde{\chi}^*_{-\boldsymbol{p}}(x)$, where $\Gamma(\boldsymbol{p})$ represents an arbitrary tensor depending on the c.m. momentum label $\boldsymbol{p}$ and $\tilde{\chi}^*_{-\boldsymbol{p}} = (i\sigma_2)\chi^*_{-\boldsymbol{p}}$. The interpolating currents associated to a definite angular momentum state L are related to irreducible representations of the tensor Γ with respect to the rotation group SO(n). The irreducible tensors are up to normalization just the spherical harmonics $Y_{LM}(n,\boldsymbol{p})$ of degree L, with $M = 1,\ldots,n_L$, that form an orthogonal basis of a n_L-dimensional space with $n_L = (2L+n-2)\,\frac{\Gamma(n+L-2)}{\Gamma(n-1)\Gamma(L+1)}$. A representation in terms of cartesian coordinates of the spherical harmonics of degree L is given by the totally symmetric and traceless tensors with L indices $T^{i_1\cdots i_L}(\boldsymbol{p})$, where the indices $i_1\ldots,i_L$ are cartesian coordinates:

$$T^{i_1\ldots i_L}(\boldsymbol{p}) = p^{i_1}\ldots p^{i_L} - \frac{\boldsymbol{p}^2}{2L+n-4}\left(\delta^{i_1 i_2}p^{i_3}\ldots p^{i_L} + \ldots\right) + \ldots\ , \quad (1)$$

and which satisfy the eigenvalue equation for the squared angular momentum operator $\boldsymbol{L}^2\, T^{i_1\ldots i_L}(\boldsymbol{p}) = L(L+n-2)\, T^{i_1\ldots i_L}(\boldsymbol{p})$. The currents with angular momenta S, P and D are *e.g.* relevant in the electromagnetic production of colored scalars from e^+e^- and $\gamma\gamma$ collisions. The use of the generalized currents built from (1) is mandatory to obtain consistent results in dimensional regularization in accordance with SO(n) rotational invariance. An instructive example regarding the computation of the nonrelativistic three-loop vacuum polarization diagram with two insertions of the Coulomb potential is explained in Ref. [18].

The interpolating currents describing the production of a fermion-antifermion pair in a spin triplet $S = 1$ state for arbitrary L (^3L_J) requires the treatment of Pauli $\boldsymbol{\sigma}$-matrices in n-dimensions. The $\boldsymbol{\sigma}$-matrices σ^i $(i = 1,\ldots,n)$ are the generators of SO(n) rotations for spin 1/2 and satisfy the Euclidean Clifford algebra $\{\sigma^i, \sigma^j\} = 2\delta^{ij}$. As for the case of the γ-matrices [19] products of $\boldsymbol{\sigma}$-matrices in arbitrary number of dimensions cannot be reduced to a finite basis, but represent an infinite set of independent structures, which can be choosen as the antisymmetrized product of $\boldsymbol{\sigma}$ matrices: $\sigma^{i_1\cdots i_m} = \sigma^{[i_1}\sigma^{i_2}\cdots\sigma^{i_m]}\,,\ m = 0,1,2,\ldots$ For $m \leq 3$ the $\sigma^{i_1\cdots i_m}$ are related to physical spin operators, with eigenvalues of $\boldsymbol{S}^2$ equal to $(0, n-1, 2, n-3)$, which reduce to the known $n = 3$ values. The $m > 3$ operators are evanescent for $n \neq 3$ (although their spin eigenvalues are non-zero). S-wave currents in an arbitrary spin state thus have the form $\psi^\dagger_{\boldsymbol{p}}(x)\, \sigma^{i_1\cdots i_m}\, \tilde{\chi}^*_{-\boldsymbol{p}}(x)$, and can arise in important processes. The structure $\sigma^{i_1 i_2 i_3}$ for example arises in fermion pair production in $\gamma\gamma$ collisions, while the evanescent operator $\sigma^{i_1\cdots i_5}$ is present in the $\bar{f}f \to 3\gamma$ annihilation amplitude. Note that the differences between the two different singlet $(m = 0, 3)$ and triplet $(m = 1, 2)$ currents correspond to evanescent operators as well.

It is well known from subleading order computations based on the effective weak Hamiltonian that one needs to consistently account for the evanescent operator structures that arise in matrix elements of physical operators when being dressed with gluons. A renormalization scheme can be adopted such that a mixing of evanescent operators into physical ones does not arise [19]. Moreover it is also known [20] that modifications of the evanescent operator basis correspond to a change of the renormalization scheme. While this does not affect physical predictions, it does affect matrix elements, matching conditions and anomalous dimensions at nontrivial subleading order. Thus precise definitions of the schemes being used have to be given to render such intermediate results useful.

In the framework of the nonrelativistic EFT these properties still apply.

However, using the velocity power counting in the EFT allows for even more specific statements. Concerning interactions through potentials, transitions between the different S-wave currents built from $\sigma^{i_1\cdots i_m}$ cannot occur because the potentials are SO(n) scalars and the currents are inequivalent irreducible representations of SO(n). Even for currents with $L \neq 0$ and for the spin-dependent spin-orbit and tensor potentials (which is all we need to consider at NNLL order) one can show that transitions between currents containing $\sigma^{i_1\cdots i_m}$ with a different number of indices cannot occur [18]. The same arguments apply to the exchange of soft gluons in vNRQCD. Concerning the exchange of ultrasoft gluons, transitions between currents containing $\sigma^{i_1\cdots i_m}$ with a different number of indices can arise, but only if the interaction is spin-dependent. The dominant among these interactions corresponds to the operator $\psi^\dagger_{\boldsymbol{p}}\sigma^{ij}k^j\psi_{\boldsymbol{p}}A^i$ and can only contribute at N^{4}LL order, which is beyond the present need and technical capabilities.

So for the S-wave currents in n dimensions one can employ either one of the two spin singlet ($m = 0, 3$) or triplet currents ($m = 1, 2$) in the EFT and the difference corresponds to a change in the renormalization scheme. This means in particular that as long as the renormalization process is restricted to time-ordered products of the currents, one can freely use three-dimensional relations to reduce the basis of the physical currents. However, once the basis of the physical currents is fixed, one has to consistently apply the computational rules in n dimensions. Moreover, one can also conclude that currents containing evanescent $\sigma^{i_1\cdots i_m}$ matrices ($m > 3$) can be safely dropped from the beginning as long as one does not need to account for spin-dependent ultrasoft gluon interactions.

Based on the latter considerations it is straightforward to construct spin-triplet currents with arbitrary L (^3L_J). They can be obtained [18] by determining irreducible SO(n) representations from products of the tensors $T^{i_1\cdots i_L}(\boldsymbol{p})$ describing angular momentum L and the spin-triplet $S = 1$ currents discussed previously. As for the case of the S-wave currents the physical basis for arbitrary spatial angular momentum is not unique due to the existence of evanescent operator structures. It is possible [18] to construct currents with fully symmetric indices equal to the total angular momentum J by using the two spin triplet operators, σ^i and σ^{ij}. There are also currents having more than J indices, which transform according to more complicated patterns of SO(n). They become equivalent in $n = 3$ to the fully symmetric currents [18] and are also appropriate to describe production of ^3L_J states.

The NLL anomalous dimensions of the currents with arbitrary spin and angular momentum configurations have been calculated in Ref. [18]. Since at LL order the currents are not renormalized, their NLL order anomalous dimensions are independent of the scheme used for the currents or the potentials. The NLL running found for the currents shows a suppression $\propto 1/(2L+1)$, which suggests that the summation of logarithms of v for the production and annihilation rates of high angular momentum states is less significant.

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